EXAM 06-11-2023

ATOMS AND MOLECULES. 18:15-20:15, # QUESTIONS: 5

YOU CAN MAKE USE (IF YOU THINK YOU NEED TO) OF THE FOLLOWING FORMULAS:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} g_J$$

1. PROBLEM 1 (?? POINTS)

A. (?? points) $^{67}\mathrm{Zn}^+$ (zinc ion) has a single valence electron in the 4s shell and a nuclear spin of $I = 5/2$. Make a schematic sketch of the energy level structure of the electronic ground state and the first excited electronic state (where the $4s$ electron is promoted to the $4p$ orbital), taking into account the spin-orbit splitting and the hyperfine structure. Assume that the spin-orbit coupling constant β and the hyperfine coupling constant A are both positive. Label all electronic levels with their F quantum number, and give the term symbols for the electronic states. Also indicate the distances between the energy levels in units of A and β.

For the s shell we have $L = 0$ and $S = \frac{1}{2}$. This means $J = \frac{1}{2}$. For the p shell we have $L = 1$ and $S = \frac{1}{2}$, so $J = \frac{1}{2}$ or $\frac{3}{2}$. By using equations 1 and 2 shown below, the energy levels of the fine and hyperfine structure can be found in terms of A and β. Figure 1 shows the level structure.

$$E_{HFS} = \frac{A}{2}\{F(F+1) - I(I+1) - J(J+1)\} \tag{1}$$

$$E_{SO} = \frac{\beta}{2}\{j(j+1) - l(l+1) - s(s+1)\} \tag{2}$$

B. (?? points) Now consider $^{66}\mathrm{Zn}^+$, $I = 0$. An external magnetic field leads to splittings and shifts of the energy levels, effectively re-arranging the ordering of the levels. In the figure below you see the energy levels of the electronically excited state (an electron is promoted from the $4s$ shell to the $4p$ shell), at two magnetic field strengths, rather weak field in case of (a) and a very strong field for (b). Explain why the energy levels are spaced and grouped as indicated for the two the panels. Where possible, use quantum numbers to label the energy levels.

Panel a) The low magnetic field has decoupled the F levels. The $^2P_{3/2}$ level is split into $m_J = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$. $^2P_{1/2}$ level is split into $m_J = \frac{1}{2}, -\frac{1}{2}$.

Panel b) The high magnetic field has decoupled the J levels. The levels from the $^2P_{3/2}$ and $^2P_{1/2}$ state are now grouped together according to $(2m_S + m_L) = -2, -1, 0, 1, 2$.

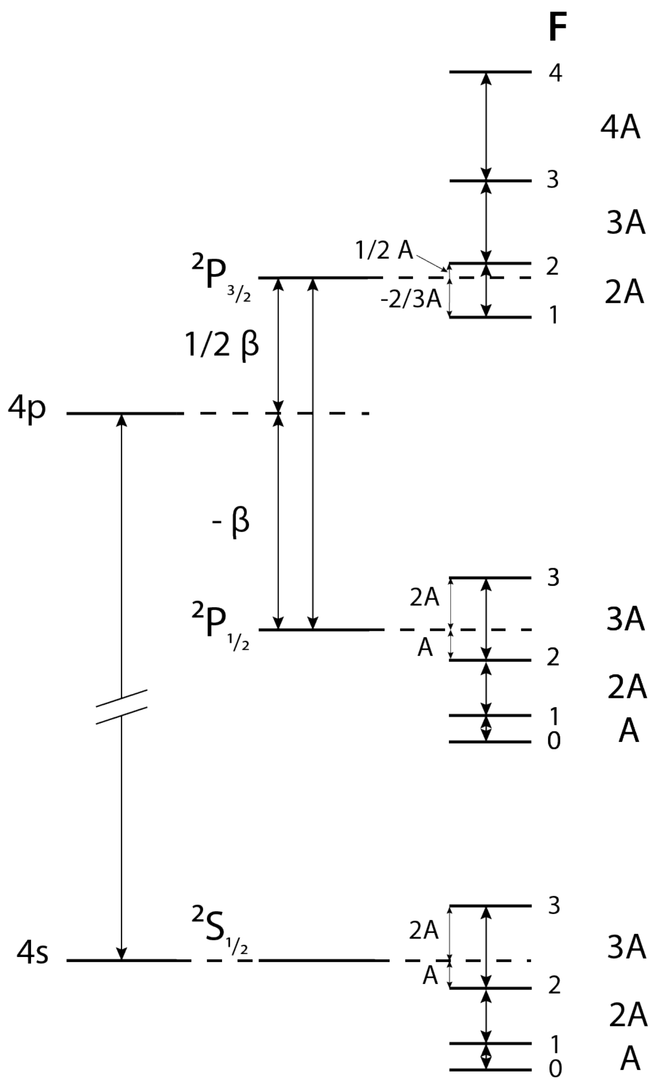


FIGURE 1. Level structure $^{67}Zn^+$

C. (?? points) $^{68}Zn^+$, like $^{66}Zn^+$, has a zero nuclear spin. What will be the difference in the frequency of the transition from the first excited state to the ground state between $^{68}Zn^+$ and $^{66}Zn^+$? Explain your answer and add a schematic sketch.

The lines will have slightly different frequencies. The heavier isotope will be at a higher frequency. See figure 3.

2. PROBLEM 2 (?? POINTS)

A. (?? points) The ground state configuration of the Selenium atom is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^4$. Which terms does this configuration give rise to? Use Hund's rules to determine the ground state.

The valence electrons are all in the 4p shell. Since it is more than half filled we can treat the $4p^4$ configuration as a $4p^2$ configuration. This means $S = 1$ and $L = 2$. We can thus

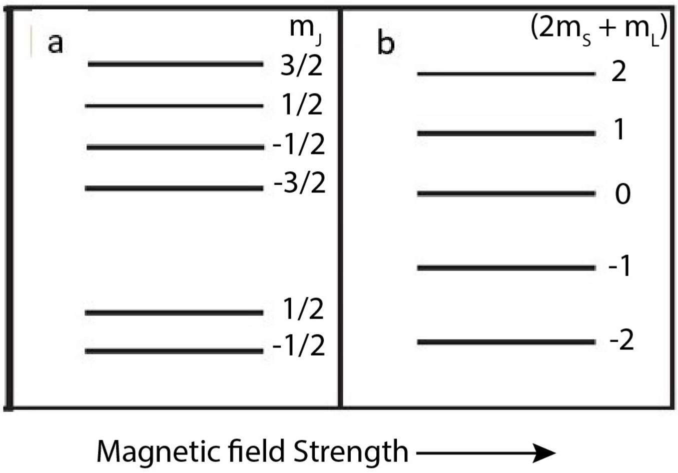
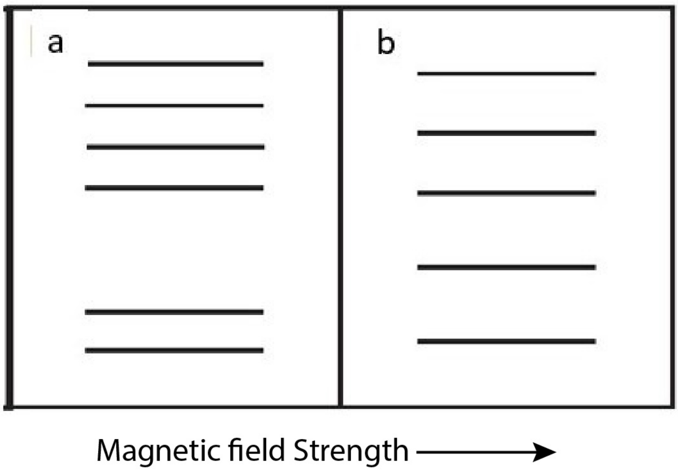


FIGURE 2



have D, P and S symbols. Because the electrons are in the same shell they are equivalent and we need to have that $L + S$ is even. So only P symbol is left. From Hund's rule 1 we have that the highest multiplicity has the lowest energy. From Hund's rule 2 we get that Highest L has lowest energy. However We can only have L=1 from the equivalence rule stated before. This then leaves us to find that the possible J levels are $J = 0, 1, 2$. From Hund's rule 3 we find that the highest J is the lowest energy because the shell is more than half full. Therefore the term symbol should be ${}^2P_2$

B. (?? points) For each of the following atomic radiative transitions, indicate whether the transition is allowed or forbidden under the electric-dipole (E1) selection rules. For the forbidden transitions, cite the selection rules which are violated, and state whether the transition could be allowed under E2 or M1 transitions.

i. Si: $3p4s\ {}^3\mathrm{P}_1 \rightarrow 3p^2\ {}^3\mathrm{P}_0$

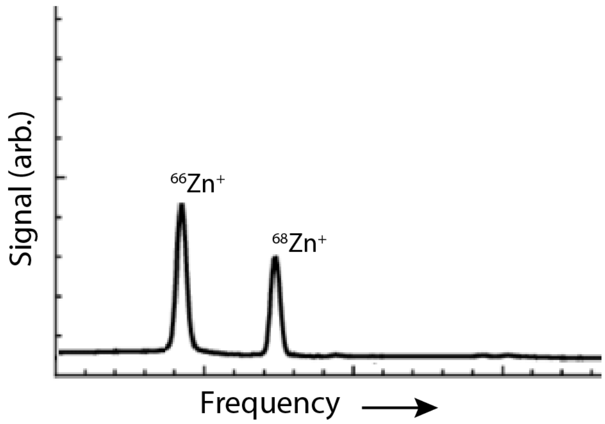


FIGURE 3

Allowed

ii. C: $2p3s\ ^3\mathrm{P}_1 \rightarrow 2p3s\ ^3\mathrm{P}_2$

Forbidden, odd to odd. Allowed as M2

iii. Cs: $5d\ ^2\mathrm{D}_{5/2} \rightarrow 6s\ ^2\mathrm{S}_{1/2}$

Forbidden, $\Delta J = 2$. Allowed as E2

iv. Mg: $3s^2\ ^1\mathrm{S}_0 \rightarrow 3s3p\ ^3\mathrm{P}_1$

Forbidden. $\Delta S = 1$

3. Interaction of resonant light with a two-level system (?? points)

A. (?? points) Explain what Rabi oscillations are and why it is much easier to observe them on an hyperfine transition compared to an electronic transition.

Rabi oscillations are coherent oscillations between two levels. They are readily observed in hyperfine transitions as opposed to electronic transitions because radio-frequency and microwave transitions have negligible spontaneous emission. Thus in most cases they evolve coherently. Electronic transitions can readily decay back to the ground state and therefore don't coherently evolve.

B. (?? points) The steady-state solutions to the optical Bloch equations for the interaction of light with a two-level atom, for times large compared to the natural lifetime of the excited state, are as follows:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \begin{pmatrix} \Omega\delta \\ \Omega\Gamma/2 \\ \delta^2 + \Gamma^2/4 \end{pmatrix}.$$

Show, using these equations, the steady state population that arises in the limit of large light intensity. Assume the light to be tuned exactly on resonance.

As the intensity increases $\Omega \rightarrow \infty$. *From the equations we see that* w *will go to zero* $w \rightarrow 0$. *We can then find that the upper level has a population of* $\frac{(1-w)}{2}$. *This will subsequently be equal to* $\frac{1}{2}$

4. Doppler-free spectroscopy (?? points)

A. (?? points) The ${}^2S_{1/2} - {}^2P_{1/2}$ transition in ^{85}Rb atoms can be excited by light at a wavelength of 795 nm. The lifetime of the excited state is 28 ns. What is the natural linewidth of this transition?

$\Delta\nu = \frac{1}{2\pi\tau} = 5.7MHz.$

B. (?? points) Give an example of a homogenious and of an inhomogenious broadening mechanism. Briefly explain these mechanisms.

A homogeneous broadening mechanism is the same for all particles in the ensemble. For example the natural linewidth, which is a property of the particles.

An inhomogeneous broadening mechanism is different for the particles in the ensemble. For example Doppler broadening, which depends on the velocity of the particles along the axis of the laserbeam

C. (?? points) Calculate the Doppler width (full-width half max) of the above transition at 300K.

First calculate the most probable speed of the gas
$v = 2230\, m/s \sqrt{\frac{T}{300\,K}\frac{1\,a.m.u.}{M({}^{85}Rb)}} = \frac{2230\, m/s}{\sqrt{85}} = 242\, m/s.$

The Doppler width is then given by
$\Delta\omega_0 = 2\sqrt{ln(2)}\frac{v}{c}\omega_0 = 2\sqrt{ln(2)}\frac{v}{\lambda} = 506MHz$

5. Slowing and cooling with lasers (?? points)

A. Show, starting from the optical Bloch equations (see below), that the maximum of the scattering force is given by $F_{max} = \frac{\hbar k}{2}$. As a reminder, the fraction of the population in the excited state is given by $\rho_{22} = \frac{1-w}{2}$.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \begin{pmatrix} \Omega\delta \\ \Omega\Gamma/2 \\ \delta^2 + \Gamma^2/4 \end{pmatrix}$$

The scattering rate is $R_{scatt} = \Gamma\rho_{22}$. Since $\rho_{22} = \frac{1-w}{2}$, look at w by the optical Bloch equations. For maximum intensity $\Omega \to \infty$, $w \to 0$. Therefore $\rho_{22}^{w\to 0} \to 1/2$. For each photon scattering we have a force equal to the photon momentum $\hbar k$. Thus we arrive at the maximum scattering force $F_{max} = \hbar k R_{scatt} = \hbar k \Gamma/2$.

B. (?? points) Consider an atom with a short excited-state lifetime (**A**) and an atom with a long excited-state lifetime (**B**), all other properties being the same. Which atom, **A** or **B**,
a) can be cooled at a higher rate using the scattering force?
b) can be cooled to a lower Doppler-limit temperature?
Explain your answer.

a) Atom A has a shorter lifetime than atom B. This means it can scatter more photons and can therefore be cooled faster.
b) The Doppler limit is a function of scattering rate: it scales linearly with scattering rate Γ. Therefore, since atom B has a lower scattering rate, it can be cooled to a lower Doppler limit than atom A.

C. (?? points)Consider an atom with a short excitation wavelength (**C**) and an atom with a long excitation wavelength (**D**), all other properties being the same.
Which atom, **C** or **D**, has the larger recoil velocity? Explain your answer.

Recoil limit $T \propto \lambda^{-2}$. Assuming all other parameters are the same between atoms C and D, this means atom D will have a lower recoil limit and atom C has a larger recoil velocity. Alternative, more hand-wavy argument. Shorter wavelength means higher energy per photon. This means that the momentum transfer upon photon absorption increases for shorter wavelength. Consequently, shorter wavelength is larger recoil velocity, so atom C has larger recoil velocity.

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b) Short excited state lifetime (A) vs long excited state lifetime

a) Atom A can be cooled at a higher rate using (B) the scattering force compared to atom B, because a shorter lifetime of the excited state means you can scatter more photons per second, which translates to a higher scattering rate.

b) The Doppler limit (eq 9.28) is a function of scattering rate: it scales linearly with scattering rate Γ. Therefore, since atom B has a lower scattering rate, it can be cooled to a lower Doppler limit than atom A.

) Short excitation wavelength (C) vs long excitation wavelength (D)

Recoil limit T (eq 9.55) $\propto \lambda^{-2}$. Assuming all other parameters are the same between atoms C and D, this means atom D will have a lower recoil limit, and atom C has a larger recoil velocity.

Alternative, more hand-wavy argument: shorter wavelength = higher energy per photon; This means that momentum transfer upon photon absorption increases for shorter wavelength light. Consequently, shorter wavelength = ~~higher~~ larger recoil velocity, so atom C has larger recoil velocity.